MA 425/525 final review problems
Version as of December 9th.

The final will be on Friday, December 15th, as scheduled on `https://www.purdue.edu/registrar/pdf/exam/current_final_exam_schedule.pdf`. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. Find the number of solutions of $z^{100} = e^z$ in $\{z \colon \operatorname{Re} z < 0\}$.

2. Let $f(z) = z^5 + 20z^2 + 22$. For each positive integer n, determine how many zeroes of f lie in the set $A_n = \{z \in \mathbb{C} \colon n - 1 < |z| < n\}$.

3. Evaluate $\int_\Gamma e^z \tan 2z \, dz$, where Γ is the contour $|z^2 - 1| = 1$, oriented as in the picture on the back page.

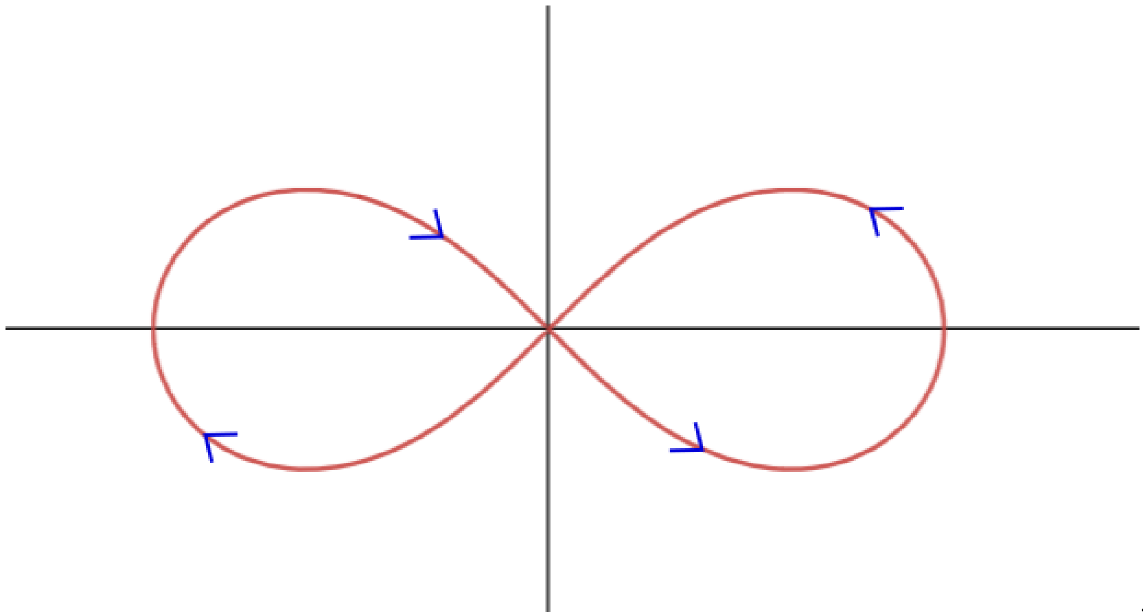
4. How many solutions does $z^{2/3} = \sqrt{3} + i$ have? Find and plot them.

5. Find integers a and b such that $(1 + i)^5 = a + bi$. Same question for $(1 + i)^{10}$, $(1 + i)^{15}$, $(1 + i)^{20}$.

6. Let a be a nonzero real number. Let Γ_1 be the segment from -1 to ai and let Γ_2 be the segment from ai to 1. Let $f(z) = \displaystyle\sum_{n=-1}^{1} z^n$. Evaluate $\displaystyle\int_{\Gamma_1} f(z)dz + \int_{\Gamma_2} f(z)dz$.

7. Let p be a real number and let $f(z) = (e^z + e^{-z})^{-1}$. What is the radius of convergence of the Taylor series of f centered at p? Same question for $f(z) = (z^2 - 2z + 5)^{-1}$ and $f(z) = (z^2 - 2z + 5)^{-1}(z^2 + 4z + 5)^{-1}$.

8. Let m and n be positive integers, and let $f(z) = (m^2 + 4)z^n + e^z$. How many zeroes does f have in $\{z \colon |z| < 1\}$ and what are their orders?



Hints:

1. See Example 3 on page 176.
2. See Example 4 on page 178.
3. Use the residue theorem after breaking Γ up into two closed loops. Pay attention to which direction the curve winds around each loop.
4. Use polar form.
5. Use polar form.
6. Choose a branch of log according to whether a is positive or negative, similar to the first two problems on the last homework.
7. Use Theorem 1 on page 123. Given p, how big can R be such that f is analytic on the disk $\{z\colon |z-p| < R\}$?
8. Use the definition on page 128 to determine how many zeroes can have order ≥ 2, without trying to find the actual zeroes.